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#### (Received 19 January 1957)

### SUMMARY

The results of measurements of the vertical distribution of airborne particles, released usually at a height of 500 ft., and sampled for periods of about 30 minutes at downwind distances of 100, 300 and 500 metres, are presented and discussed.

At all distances the frequency distributions of the particle elevation with respect to the site of release are closely similar in shape and size to the frequency distribution of wind inclination at the site of release. This is interpreted as showing that, despite the uncorrelated effects of small eddies, high correlation was maintained in the motion of a particle, for periods of 1 minute or more, by the dominant action of persistent eddies which contributed heavily to the turbulent energy. In contrast to this result, the wind observations showed that the Eulerian auto-correlation coefficient  $(R_t)$  fell to about 0.2 in 10 seconds.

### 1. INTRODUCTION

The turbulence which occurs in the atmosphere clear of the ground is often approximately steady and homogeneous. Under these conditions, the diffusion of particles released at a fixed point is specified by the wellknown Taylor (1921) relation

$$\overline{z^2} = 2\overline{w'^2} \int_0^T \int_0^t R_{\xi} d\xi dt.$$
 (1)

Here z is the vertical displacement of particles from their mean vertical position after travelling a time T, w' is the deviation from the mean of the vertical component of the velocity of the particles, and  $R_{\xi}$  is the correlation coefficient between the vertical velocity at one instant and the velocity of the same particle at a time  $\xi$  later.  $R_{\xi}$  is usually referred to as the Lagrangian correlation coefficient. In practice  $R_{\xi}$  is very difficult, if not impossible, to measure effectively and in problems of atmospheric diffusion progress has depended (see Sutton 1932, 1934) on specifying intuitively or empirically some general relation between  $R_{\xi}$  and measurable quantities. One empirical approach, however, which does not seem to have been exploited to any extent, is to use experimental measurements of diffusion to obtain some indications of the properties of the Langrangian correlation coefficient in comparison with those of the corresponding, measurable, Eulerian coefficient

 $R_{t'}$ , that is, the correlation between the velocity of the air at a point at one instant and that at the same point at time t' later. The simplest possible type of comparison would be to evaluate

$$\frac{1}{T^2}\int_0^T\int_0^t R_{t'} dt'dt$$

and compare it, as a function of T, with the corresponding double integral from (1). After commencing the present investigation the writers' attention was drawn to some recent work by Mickelsen (1955) in which precisely this line of approach has been followed in studying diffusion in homogeneous turbulence in a wind tunnel.

The present paper describes experiments in which the vertical distribution of airborne particles, released in the atmosphere at a position remote from the ground, was examined at various distances downwind up to 500 m, using simple lightweight apparatus carried on the cables of captive balloons. Simultaneously, measurements of the fluctuations in the inclination of the wind at the site of release were made, in order effectively to specify w' and  $R_{t'}$ , using a balloon-borne gustiness recorder described previously by Jones (1956).

### 2. Equipment and technique

The particles used in these experiments, spores of Lycopodium, were chosen because they are large enough to impact with reasonable efficiency on cylinders of convenient diameter, and can thus be collected from the air without the aid of elaborate sampling apparatus. This method of sampling was used successfully by Gregory (1951) in wind tunnel experiments. As these spores come from a type of moss usually found in mountainous districts, it was unlikely that they would be present naturally in the atmosphere over downland near Salisbury, a fact confirmed during the course of the experiments. Their characteristic size  $(30\mu \text{ diameter})$ , shape and colour enable them to be distinguished without undue difficulty from other particles suspended in the atmosphere, while their terminal velocity (about 2 cm/sec) seems low enough not to affect their scattering in a turbulent atmosphere.

A dispenser, shown in figure 1 (plate 1), emitted the particles in a quasi-continuous manner, at an average rate of 3 gm/min. The spores are gravity fed from a hopper (of dimensions  $17 \times 6 \times 1$  in.) onto a toothed wheel rotating in a Perspex block, from which they emerge into the air and onto the blades of a small high-speed fan which serves to break up aggregates of spores. The spores are emitted in discrete puffs but the interval between puffs is only about  $\frac{1}{4}$  sec. In practice, the dispenser was mounted on the cable of a large captive balloon, usually at about 500 ft. above the ground, and operated for 30 min or so at a time. Power for the motor driving the toothed wheel and fan was supplied from a source on the ground via lightweight electric cables.

J. S. Hay and F. Pasquill, Diffusion from a fixed source at a height of a few hundred feet in the atmosphere, Plate 1.



Figure 1. The particle dispenser and wind inclination instrument on balloon cable.

The apparatus for collecting the particles consisted simply of 5 cm lengths of  $\frac{1}{4}$  in. glass tubing, of which some 4 cm were covered with an adhesive similar to that described by Gregory (1951). Cylinders of this size are easily handled and examined and, according to Gregory, their collection efficiency\* for *Lycopodium* spores in winds of more than  $3\cdot 3$  m/sec exceeds 40% and does not vary appreciably with wind speed. Throughout each emission, the adhesive cylinders were held into wind by small vanes spaced along the cable of a small captive balloon so that the central vane was about level with the dispenser. This balloon was positioned downwind of the balloon carrying the dispenser, at a distance of 100, 300 or 500 m, and was moved as necessary to allow for any substantial changes in wind direction during the emission.

In the subsequent assessment of the samples collected, twenty sections of each cylinder were examined by a microscope having a field of view about 0.12 cm square and the number of particles in each section was counted. These numbers were found to follow a Poisson distribution so that the mean value, which was taken to be a measure of the deposit on a cylinder, had a standard error which was easily calculated.

Mounted immediately above the dispenser (see figure 1) was the instrument for the simultaneous continuous recording of the fluctuations of wind inclination and of mean wind speed (Jones 1956). Thermistors were also mounted on the cable and connected to recording apparatus on the ground in order to give the temperature difference between the level of the dispenser and a point 200 ft. below. The temperature difference between 23 ft. and 4 ft. above the ground was obtained from permanent apparatus located nearby. To allow corrections for the vertical movement of both balloons to be applied, observations of the elevations of the dispenser and central sampling vane were made by means of a theodolite. The time interval between observations was usually 10 sec in the case of the dispenser and 1 min in the case of the central sampling vane.

### 3. REDUCTION AND PRESENTATION OF OBSERVATIONS

## (a) Particle distributions

Of the 19 experiments which were carried out, it was evident that in 9 cases the cloud of particles had not been adequately sampled, due to insufficient spacing or incorrect downwind positioning of the sampling cylinders, and these were not considered further. In the remaining 10 cases, to facilitate comparison with the measured wind inclinations at the site of release, the vertical distribution of particles was expressed in terms of their angular elevations with respect to the dispenser. The particle deposit at each level was taken to be representative of the interval

\* The collection efficiency is the number of particles impacting on the cylinder, expressed as a percentage of the number which would have passed through the same cross-sectional area if the cylinder had not been there. For a given particle size, the collection efficiency depends on the diameter of the cylinder, the speed of the air stream and the type of adhesive used. centred on the cylinder and extending mid-way to the adjacent sampling levels. Cumulative deposits, obtained by addition of successive individual values, and expressed as a percentage of the total, are plotted against elevation on probability graph paper in figures 2 and 3.



Figure 2. Distributions of particle elevation (x) and wind inclination (③) for experiments 4, 7, 8, 16 and 17. Ordinate scale is given for experiment 4. Scales in other cases are similar with zero lines indicated by arrows.

The observed spread of particles contains a contribution arising from the relative vertical motion of the dispenser and sampling array due to balloon movements. This contribution depends on the difference,  $\Delta H$ , between the height of a datum point on the sampling array at time T and the height of the dispenser at time T-t, where t is the time of travel of the particle. It seems unlikely that there could have been a high degree of correlation between  $\Delta H$  and the true vertical displacement of the particle with respect to its point of origin. This supposition was confirmed by examining the relation between  $\Delta H$  and  $\phi$ , where  $\phi$  is the wind inclination at the dispenser position at time T-t, for experiment no. 8, which represents the worst case in the sense that the variations of  $\Delta H$  relative to the apparent vertical displacements of particles were largest. The usual statistical rule for deriving the variance of the difference of two independent quantities may therefore be applied. If  $\sigma_s^2$  and  $\sigma_a^2$  are



Figure 3. Distributions of particle elevation (x) and wind inclination (⊙) for experiments 12, 13, 14, 15 and 19. Ordinate scale is given for experiment 12. Scales in other cases are similar with zero lines indicated by arrows.

respectively the variances of  $\Delta H$  and the apparent vertical displacements of the particles, then  $\sigma_p^2$ , the variance of the true vertical displacements, is given by  $\sigma_p^2 = \sigma_a^2 - \sigma_s^2$ .

In practice, these quantities were expressed as elevations with respect to the dispenser. The values of  $\Delta H$  were obtained from the theodolite

observations, the time t being estimated from the wind speed measured over periods of a few minutes.

### (b) Distributions of wind inclination

From the gustiness records of these same 10 experiments, values of the wind inclination  $\phi$  were read off to the nearest  $\frac{1}{2}^{\circ}$  at  $2\frac{1}{2}$ -sec intervals. Because of the cramped nature of the records, it was frequently possible to estimate only upper and lower limits for the value of  $\phi$ , and in such cases the mean of these limits was taken as the appropriate value. Corrections for deflections of the wind-inclination vane, which arose from the movement of the supporting balloon rather than from the turbulent motion of the air, were then applied as follows.

If the elevation  $\alpha$  of the wind-inclination vane as observed by the theodolite is small, and changes by an amount  $\Delta \alpha$  in a time  $\Delta t$  (usually 10 sec), the spurious component of wind inclination over this time interval is approximately  $\phi_1 = -i\Delta\alpha/u\Delta t$  where l is the horizontal distance from the theodolite to the vane (about 3000 m) and u is the indicated horizontal wind speed over the interval. The error is principally due to the neglect of the horizontal components of the eddy motion and the motion of the balloon. In evaluating the expression, it was assumed that values of u averaged over a period of 2 or 3 min applied to the component intervals  $\Delta t$ . Instantaneous values of  $\phi_1$  at times corresponding to the readings of  $\phi$  were then estimated to the nearest  $\frac{1}{2}^\circ$  from these short period means by graphical interpolation, and used to correct the values of  $\phi$ .

The corrected values of wind inclination, that is,  $\phi - \phi_1$ , from each experiment were formed into a frequency distribution based on class intervals which were one degree wide and centred on integral values of inclination, the frequencies of the intermediate half-degree values being divided equally between adjacent intervals. For comparison with the particle distributions, cumulative frequency distributions were formed for the range of elevations covered by the sampling cylinders. This procedure necessarily excluded the occasional extreme values of inclination, which were mainly positive in sign, that is, indicative of upward moving currents. These distributions are shown together with the corresponding particle distributions in figures 2 and 3, the cumulative frequencies being plotted against the upper limits of the respective class intervals.

## (c) Meteorological and other conditions

The dates and times of the various experiments, and the distances and heights involved, are given in table 1 together with wind speed and local and surface temperature gradient measurements. Mean wind speeds at the height of release ranged from 6.1 to 9.5 m/sec and the temperature gradient in the lower layers ranged from slight lapse to slight inversion.

		1			
Experiment Number	4	7	8	12	13
Date (1955)	16 Sept.	26 Sept.	26 Sept.	4 Oct.	4 Oct.
Time (G.M.T.)	0828-0844	1436-1506	1606-1636	1521-1601	1636-1716
Distance of sampling					
(m)	100	100	100	300	300
Height of dispenser					
above ground (m)	143	147	255	140	143
Wind speed (m/sec)	6.7	8.0	7.4	7.7	7.3
Local temperature					
gradient		0 <b>∙9</b> 0Γ		0·66 <b>Γ</b>	0.62Γ
Surface temperature					
gradient (23'-4')					
(°F)	0.6	+0.5	0	-0.4	+0.6
Cloud amount and	🖁 As	<b>∦</b> Sc 5000	$\frac{5}{8} - \frac{7}{8}$ Sc	🛔 Sc 5000	🖁 Sc 5000
height (ft.)	₿ Sc 2500	🕴 Cu 3000	3500	<b>å Cu 3000</b>	🛔 Cu 3000
0	v	Ŭ		-	U U
			<u>.                                    </u>		
	1		1		
Experiment Number	14	15	16	17	19
Experiment Number Date (1955)	14 13 Oct.	15 14 Oct.	16 17 Oct.	17 17 Oct.	19 25 Oct.
Experiment Number Date (1955) Time (G.M.T.)	14 13 Oct. 1522–1552	15 14 Oct. 1425–1455	16 17 Oct. 1151–1221	17 17 Oct. 1430–1500	19 25 Oct. 1517–1557
Experiment Number Date (1955) Time (G.M.T.) Distance of sampling	14 13 Oct. 1522–1552	15 14 Oct. 1425–1455	16 17 Oct. 1151–1221	17 17 Oct. 1430–1500	19 25 Oct. 1517–1557
Experiment Number Date (1955) Time (G.M.T.) Distance of sampling (m)	14 13 Oct. 1522–1552 300	15 14 Oct. 1425–1455 300	16 17 Oct. 1151–1221 500	17 17 Oct. 1430–1500 500	19 25 Oct. 1517–1557 300
Experiment Number Date (1955) Time (G.M.T.) Distance of sampling (m) Height of dispenser	14 13 Oct. 1522–1552 300	15 14 Oct. 1425–1455 300	16 17 Oct. 1151–1221 500	17 17 Oct. 1430–1500 500	19 25 Oct. 1517–1557 300
Experiment Number Date (1955) Time (G.M.T.) Distance of sampling (m) Height of dispenser above ground (m)	14 13 Oct. 1522–1552 300 145	15 14 Oct. 1425–1455 300 154	16 17 Oct. 1151–1221 500 150	17 17 Oct. 1430–1500 500 151	19 25 Oct. 1517–1557 300 129
Experiment Number Date (1955) Time (G.M.T.) Distance of sampling (m) Height of dispenser above ground (m) Wind speed (m/sec)	14 13 Oct. 1522–1552 300 145 6:3	15 14 Oct. 1425–1455 300 154 6:1	16 17 Oct. 1151–1221 500 150 7·3	17 17 Oct. 1430–1500 500 151 6:8	19 25 Oct. 1517–1557 300 129 9:5
Experiment Number Date (1955) Time (G.M.T.) Distance of sampling (m) Height of dispenser above ground (m) Wind speed (m/sec) Local temperature	14 13 Oct. 1522–1552 300 145 6·3	15 14 Oct. 1425–1455 300 154 6·1	16 17 Oct. 1151–1221 500 150 7·3	17 17 Oct. 1430–1500 500 151 6·8	19 25 Oct. 1517–1557 300 129 9·5
Experiment Number Date (1955) Time (G.M.T.) Distance of sampling (m) Height of dispenser above ground (m) Wind speed (m/sec) Local temperature gradient	14 13 Oct. 1522–1552 300 145 6·3 0-825	15 14 Oct. 1425–1455 300 154 6·1 0·86F	16 17 Oct. 1151–1221 500 150 7·3 0.87[	17 17 Oct. 1430–1500 500 151 6·8 0·76Г	19 25 Oct. 1517–1557 300 129 9·5
Experiment Number Date (1955) Time (G.M.T.) Distance of sampling (m) Height of dispenser above ground (m) Wind speed (m/sec) Local temperature gradient Surface temperature	14 13 Oct. 1522-1552 300 145 6·3 0·82Γ	15 14 Oct. 1425–1455 300 154 6·1 0·86Γ	16 17 Oct. 1151–1221 500 150 7·3 0·87Г	17 17 Oct. 1430–1500 500 151 6·8 0·76Γ	19 25 Oct. 1517–1557 300 129 9·5 —
Experiment Number Date (1955) Time (G.M.T.) Distance of sampling (m) Height of dispenser above ground (m) Wind speed (m/sec) Local temperature gradient Surface temperature gradient (23'-4')	14 13 Oct. 1522-1552 300 145 6·3 0·82Γ	15 14 Oct. 1425–1455 300 154 6·1 0·86Γ	16 17 Осt. 1151–1221 500 150 7·3 0·87Г	17 17 Oct. 1430–1500 500 151 6·8 0·76Γ	19 25 Oct. 1517-1557 300 129 9.5 —
Experiment Number Date (1955) Time (G.M.T.) Distance of sampling (m) Height of dispenser above ground (m) Wind speed (m/sec) Local temperature gradient Surface temperature gradient (23'-4') (°F)	14 13 Oct. 1522–1552 300 145 6·3 0·82Г 0·4	15 14 Oct. 1425–1455 300 154 6·1 0·86Γ 0·6	16 17 Oct. 1151–1221 500 150 7·3 0·87Г 0·7	17 17 Oct. 1430–1500 500 151 6·8 0·76Γ –0·3	19 25 Oct. 1517-1557 300 129 9·5  +0·2
Experiment Number Date (1955) Time (G.M.T.) Distance of sampling (m) Height of dispenser above ground (m) Wind speed (m/sec) Local temperature gradient Surface temperature gradient (23'-4') (°F) Cloud amount and	$ \begin{array}{r}     14 \\     13 \text{ Oct.} \\     1522-1552 \\     300 \\     145 \\     6\cdot3 \\     0\cdot82\Gamma \\     \hline     -0\cdot4 \\     \frac{2}{3} \text{ CiCs} \end{array} $	$ \begin{array}{c} 15\\ 14 \text{ Oct.}\\ 1425-1455\\ 300\\ 154\\ 6\cdot1\\ 0\cdot86\Gamma\\ -0\cdot6\\ 5 \text{ Sc } 2500\\ \end{array} $	16 17 Oct. 1151-1221 500 150 7·3 0·87Γ 0·7 ≹ AcAs	17 17 Oct. 1430–1500 500 151 6·8 0·76Γ 	$ \begin{array}{r}     19 \\     25 \text{ Oct.} \\     1517 - 1557 \\     300 \\     129 \\     9 \cdot 5 \\     - \\     + 0 \cdot 2 \\     \frac{2}{3} - \frac{4}{3} \text{ Sc} \end{array} $
Experiment Number Date (1955) Time (G.M.T.) Distance of sampling (m) Height of dispenser above ground (m) Wind speed (m/sec) Local temperature gradient Surface temperature gradient (23'-4') (°F) Cloud amount and height (ft.)	$ \begin{array}{c} 14\\ 13 \text{ Oct.}\\ 1522-1552\\ 300\\ 145\\ 6\cdot3\\ 0\cdot82\Gamma\\ -0\cdot4\\ \frac{2}{8} \text{ CiCs}\\ \frac{1}{8}-\frac{2}{8} \text{ Cu} \end{array} $	$ \begin{array}{c} 15\\ 14 \operatorname{Oct.}\\ 1425-1455\\ 300\\ 154\\ 6.1\\ 0.86\Gamma\\ 0.86\Gamma\\ \begin{array}{c} -0.6\\ & \operatorname{Sc} 2500\\ \end{array} $	16 17 Oct. 1151-1221 500 150 7·3 0·87Γ 	17 17 Oct. 1430–1500 500 151 6·8 0·76Γ -0·3 <sup>§</sup> CuSc 3000	$ \begin{array}{r}     19 \\     25 \text{ Oct.} \\     1517 - 1557 \\     300 \\     129 \\     9 \cdot 5 \\     \\     +0 \cdot 2 \\     \frac{2}{8} - \frac{4}{8} \text{ Sc} \\     3000 \\     3000 \end{array} $
Experiment Number Date (1955) Time (G.M.T.) Distance of sampling (m) Height of dispenser above ground (m) Wind speed (m/sec) Local temperature gradient Surface temperature gradient (23'-4') (°F) Cloud amount and height (ft.)	$ \begin{array}{c} 14\\ 13 \text{ Oct.}\\ 1522-1552\\ 300\\ 145\\ 6\cdot3\\ 0\cdot82\Gamma\\0\cdot4\\ \frac{2}{5} \text{ CiCs}\\ \frac{1}{3}-\frac{3}{5} \text{ Cu}\\ 2500 \end{array} $	$ \begin{array}{r} 15\\ 14 \text{ Oct.}\\ 1425-1455\\ 300\\ 154\\ 6\cdot1\\ 0\cdot86\Gamma\\ & -0\cdot6\\ & \text{s Sc 2500}\\ \end{array} $	$ \begin{array}{r} 16\\ 17 \text{ Oct.}\\ 1151-1221\\ 500\\ 150\\ 7\cdot3\\ 0\cdot87\Gamma\\ -0.7\\ \frac{3}{4} \text{ AcAs}\\ \frac{3}{8}-\frac{3}{2} \text{ CuSc}\\ 3000 \end{array} $	$     \begin{array}{r}       17 \\       17 \text{ Oct.} \\       1430-1500 \\       500 \\       151 \\       6.8 \\       0.76\Gamma \\       -0.3 \\       \frac{-0.3}{\$ \text{ CuSc}} \\       3000 \\     \end{array} $	$ \begin{array}{r}     19 \\     25 \text{ Oct.} \\     1517 - 1557 \\     300 \\     129 \\     9 \cdot 5 \\     \\     +0 \cdot 2 \\     \frac{2}{8} - \frac{4}{8} \operatorname{Sc} \\     3000 \\ \end{array} $

Table 1. Summary of meteorological and other conditions. The local temperature gradient was obtained from measurements over a height interval of 200 ft. near the dispenser.  $\Gamma = dry$  adiabatic lapse rate.

# 4. The observed relation between the distributions of particle elevation and wind inclination

The frequency distributions of particle elevation and wind inclination collected in figures 2 and 3 show three features of interest. Firstly there is the relation between the median values of the two distributions. For seven of the experiments there is agreement to within 1°, in one experiment the particle elevation is higher by 3°, while in the remaining two, it is lower by 2°. The mean of the differences in median values, in the sense wind inclination minus particle elevation, is  $+0.2^{\circ}$  with a standard deviation of  $1.5^{\circ}$ . In other words, there is no significant indication of a declination of the trajectories of the particles with respect to the initial air flow. However, it is noteworthy that this value  $+0.2^{\circ}$  is very close to that which would follow from the expected terminal velocity of 2 cm/sec in a wind of about In individual experiments the gravitational fall is evidently 7 m/sec.negligible in comparison with the unknown effects of incomplete sampling (in a statistical sense), and of systematic features such as topographical control on the general flow pattern or variations of eddy structure with height.

F.M.

A second feature is that the graphs in figures 2 and 3 are roughly linear, indicating an approximation to Gaussian form in the distributions of both wind inclination and particle elevation. This is not a particularly novel conclusion concerning the wind data-except in so far as it refers to wind inclinations at an unusually elevated site-for it is widely thought that turbulent components will usually be so distributed, apart from anisotropic motions associated with well-developed convective action. In this latter respect it is to be borne in mind that the restriction of our comparison of wind and particle distributions to the height range covered by the sampling apparatus has necessarily excluded the occasional extreme inclinations, most of which are a reflection of this anisotropic feature. The main interest is rather in the approximately Gaussian form of the particle distributions, for here it is to be remembered that the measurements are not made at a single level of say 500 ft., but over a layer which may extend between 200 and 800 ft. and which will contain systematic variations of wind and eddy structure with height. Inspection of the individual distributions does not suggest any consistent departure from Gaussian form and it is interesting to note that there is no outstanding difference in this respect between results obtained at different distances from the source.

The third and most interesting feature is the similarity in the dimensions of the wind and particle distributions, though before any realistic comparison can be made it is necessary to correct for the relative vertical displacements of the dispenser and the sampling array, as discussed previously. Values of  $\sigma_{a}$ , the standard deviation of the apparent elevations of the particles, were extracted from the distributions in figures 2 and 3 by reading off the elevations at 2 and 98%, that is, those enclosing 4.12 times the standard deviation in a Gaussian distribution. A similar process was used to find  $\sigma_w$ , the standard deviations of wind inclination. The standard deviations,  $\sigma_s$ , of the elevations due to the relative vertical displacements were computed The values of  $\sigma_a$  and  $\sigma_s$ , the resulting values of  $\sigma_p$ , and the directly. corresponding values of  $\sigma_w$ , are collected in table 2 below. Examination of the statistics of sampling indicates that the standard errors in the individual values of particle deposit range from a few per cent for high deposits to greater than 20% for the very low deposits. The net error in  $\sigma_a$  due to these errors is estimated to be of the order of 3%. In the case of  $\sigma_w$  the main error is due to the uncertainty in the correction applied for the movement of the balloon. An upper limit to this error has been estimated for one case (no. 17) and found to be of the order of 4%.

With the exception of nos. 4 and 19 there is a remarkable degree of similarity between the values of  $\sigma_p$  and  $\sigma_w$ , irrespective of the distance of travel and the conditions of wind and atmospheric stability prevailing. The odd result in the case of no. 19 is clearly associated with certain subsidiary peaks in the particle distribution, which occur at a height well below the main peak and for which there appear to be insufficient corresponding large negative values of wind inclination. No. 4 was carried out in pre-frontal conditions with drizzle setting in, and is a special case in that an abrupt

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reduction in gustiness occurred about half-way through the experiment. The observation of a distribution of particles which is narrow relative to the distribution of wind inclination could be due to unrepresentative sampling, since the positioning of the sampling array with respect to the cloud of particles was probably controlled more successfully in the second half of the experiment than in the first. This idea is supported by the very high particle deposits that were observed over a sampling time of only 16 minutes.

Expt of		Particle elevation			Wind		ū	$\Delta T^{\circ} F$	
no. travel (m)	$\sigma_{s}$	$\sigma_a$ (degrees)	$\sigma_p$	ation $\sigma_w$ (degrees)	$\sigma_p/\sigma_w$	(m/sec)	(23 ft.– 4 ft.)		
4	100	1.2	2.1	1.7	3.1	0.55	6.7	-0.6	
7	100	1.3	3.9	3.7	3.0	1.23	8∙0	+0.2	
8	100	2.2	<b>4</b> ·1	3.5	2.8	1.25	7.4	0	
12	300	0.7	5.0	4.9	4.6	1.07	7.7	-0.4	
13	300	0.5	3.9	3.9	3.8	1.03	7.3	+0.6	
14	300	1.0	<b>4</b> ∙2	<b>4</b> ∙1	<b>4</b> ∙0	1.03	6.3	-0.4	
15	300	0.2	4.9	4.9	4.9	1.00	6.1	-0.6	
16	500	0.5	3.0	3.0	3.2	0.94	7.3	0.7	
17	500	0.3	2.9	2.9	2-7	1.07	6.8	-0.3	
19	300	0.9	6.1	6.0	4∙0	1.50	9.5	+0.5	
					<u> </u>				



The original data have been independently examined by a statistician. That the distribution was approximately Gaussian, in one case which appeared doubtful, was confirmed by the application of a formal test. In the subsequent treatment, therefore, the data were assumed to be of Gaussian form. Variances of apparent particle elevation  $(\sigma_a^2)$  and of wind inclination  $(\sigma_w^2)$  were calculated directly from the observations. The values of  $\sigma_a^2$  were then corrected as before to give  $\sigma_p^2$ . (The values of  $\sigma_p$ and  $\sigma_w$  obtained in this fashion differed only slightly from those presented in table 2.) Owing to the large number of degrees of freedom involved, the equality of  $\sigma_p^2$  and  $\sigma_w^2$  could not conveniently be tested from the available statistical tables of variance ratio. Instead, a form of t-test was applied to the differences of the variances. Of the ten experiments, six (nos. 12, 13, 14, 15, 16, 17) showed no significant difference, while for the remaining four (nos. 4, 7, 8, 19) the differences were found to be highly significant. However, only in no. 4 is  $\sigma_p$  substantially less than  $\sigma_w$ , and this is the experiment which has already been discussed as possibly anomalous.

5. INTERPRETATION OF THE RESULTS IN RELATION TO EDDY STRUCTURE

As stated in the Introduction to this paper, the fundamental object of the present investigation was to obtain indirectly some insight into the Lagrangian features of the eddy structure, that is, variations which are related to time in the sense of following the motion, in comparison with Eulerian features, which are related to time at fixed points in space. From Taylor's expression for the scattering of particles (equation (1)), it immediately and necessarily follows that a linear variation of standard deviation with time (in our notation,  $\sigma_p = \sigma_w$ ) is held as long as the Lagrangian correlation coefficient remains close to unity. For such a time, the particles travel in straight lines with the velocity imparted to them at the moment of release. It should be noted, however, that the departure from the linear variation of  $\sigma_p$  is not sensitively dependent on the initial fall in the correlation coefficient. For example, if this fall is at an exponential rate, the value of  $\sigma_p/\sigma_w$  will still be as much as 0.86 when the correlation coefficient has fallen to about 0.4.

The present results, in showing equality between  $\sigma_w$  and  $\sigma_p$  for distances up to 500 m, imply that the diffusive spread of the particles may be described by equation (1) with  $R_{\xi}$  maintained at a relatively high value for at least 1 min. This period of maintenance of Lagrangian correlation is considerably longer than that which is evident in the Eulerian or auto-correlation coefficient computed from corrected values of the wind inclination\* at  $2\frac{1}{2}$ -sec intervals, as can be seen from the two cases quoted below in table 3. These figures are the means of three values obtained from analysis of three separate 8-min periods contained in each of the two experiments.

Time lag (sec)		$2\frac{1}{2}$	5	$7\frac{1}{2}$	10	15	20	30
Correlation coefficient	Expt. no. 13	0.69	0.40	0.19	0.09	-0.01	0.01	0.02
	Expt. no. 17	0.61	0.40	0.29	0.23	0.20	0.14	0.04

Table 3. Values of auto-correlation coefficient for wind inclination.

The simplest interpretation of the present measurements of Lagrangian correlation is that, despite the rapid Eulerian variations, the particles travel in virtually straight lines for substantial periods. It is clear from practical experience, however, that this interpretation cannot be literally correct. Indeed, it is obvious from visual observations of a puff of smoke moving with the wind that the trajectories of individual smoke particles are far from linear, and are in fact rapidly distorted mainly by turbulent motions on a scale smaller than that of the puff itself.

\* For the purpose of the arguments which are ultimately developed in the present paper it is probably unnecessary to give detailed consideration to the differences between the auto-correlation for wind inclination  $(R_{t'}(\phi))$  and that for the vertical component of eddy velocity  $(R_{t'}(w))$ . However, an analysis of this feature has confirmed that small values of  $R_{t'}(\phi)$ , of the order of 0.1, are necessarily associated with similarly small values of  $R_{t'}(w)$ . The explanation of the above paradox clearly lies in the form of the spectrum of eddy motion existing in the atmosphere. The trajectory of a single particle may be regarded as composed of a series of harmonic oscillations. If the oscillations of low and high frequencies have respectively large and small amplitudes, the particle will tend to follow a path which contains small-scale irregularities about a large-scale relatively smooth curve. The latter will be virtually straight over appreciable distances, and will therefore lead to high values of the Lagrangian correlation coefficient for the particle.

A more general analytical discussion of this effect of the Lagrangian spectrum of turbulence has been given by Batchelor (1949), who demonstrated that the statistical dispersion of a single fluid particle is determined initially by all frequencies (of the particle velocity) in proportion to their energies, but that at greater distances from the point of release the effects of the lower frequencies become relatively more important. The point which is brought out by the present experimental results in the atmosphere is that, despite the uncorrelated effects of small eddies, high correlation may be maintained in the motion of a particle by the action of persistent eddies which contribute heavily to the total energy of turbulence. It may at first sight seem surprising that, in contrast to this result, Edinger's (1951) measurements, with soap bubbles in an isothermal layer at a height of 1000 ft., indicate a Lagrangian correlation coefficient falling to 0.5 in about 7 sec. However, this is presumably due to the fact that Edinger's observations were made over a total time of only a few seconds, and therefore could not include the effects of any large persistent eddies, even if these existed in the conditions of his experiments.

The difference in the persistence of correlation in the Eulerian and Lagrangian systems is presumably associated mainly with the fact that in the Eulerian measurements the pattern of eddy motion is being swept more or less rapidly past the fixed observing point. The effect has previously been observed in wind tunnel turbulence, and, indeed, in an investigation similar in some respects to the present one, Mickelsen (1955) has recently compared the diffusive spread of a gas emitted from a point in wind tunnel flow with that calculated from equation (1) with the Lagrangian correlation coefficient replaced by measured Eulerian values. Equality in the observed and computed forms of  $\sigma^2/2\overline{v^2}T^2$ , where v is the cross-stream component of eddy velocity, was found to occur when T (Lagrangian) was approximately 3 times T (Eulerian). This relation held for a range of values of  $\sigma^2/\overline{v^2}T^2$  from 1 to 0.4. The data did not permit evaluation at lower values of  $\sigma^2/\overline{v^2}T^2$  but the trend appeared to be in the direction of a rapid increase in the ratio of the 'Lagrangian' to the 'Eulerian' time.

In contrast to Mickelsen's experiments, ours do not extend into the region of non-linear relation between  $\sigma^2$  and  $t^2$ , and a similar evaluation of the ratio of the 'Langrangian' to the 'Eulerian' time for equality of the above forms is accordingly not possible. However, substitution in equation (1) of the auto-correlation data given in table 3 leads to a

 $(\sigma, t)$ -relation which begins to depart significantly from linearity (i.e.  $\sigma^2/\overline{w'^2}t^2 \Rightarrow 0.8$ ) for t between 5 and 10 sec, whereas in the observed values a similar order of departure from linearity is not evident even at 70 sec. In other words, this suggests that for equal values of  $\sigma^2/\overline{w'^2}t^2$ , the ratio of the 'Lagrangian' to the 'Eulerian' time is not less than about 10:1.

In the observational study of atmospheric turbulence there have recently been some quite novel measurements made by Gifford (1955), in an attempt to compare the Lagrangian and Eulerian characteristics directly at a height of 300 ft. Gifford uses the trajectories of 'no-lift' balloons from which to extract vertical velocities of a Lagrangian nature, and compares these on an energy spectrum basis with Eulerian values from a fixed installation. The interesting feature of these results is the suggestion that the Lagrangian spectrum is generally displaced to the *low* frequency side of the Eulerian spectrum, with the respective frequencies of peak energy in the ratio of 1:3approximately.

It seems clear, therefore, that our conclusion regarding the difference in Lagrangian and Eulerian persistence of eddy velocity is not inconsistent qualitatively with the results of Mickelsen and Gifford.

Acknowledgment is made to the members of the Ministry of Supply Establishments at Porton who assisted in various aspects of the work. The large captive balloon was provided and manned by R.A.F., Cardington. Acknowledgment is also made to the Director of the Meteorological Office, the Chief Scientist of the Ministry of Supply and the Controller of H.M. Stationery Office for permission to publish this paper. (Crown Copyright Reserved.)

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